# UTILITARIANISM AND INFINITE UTILITY

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## I. Introduction

Traditional act utilitarianism judges an action permissible just in case it produces as much aggregate utility as any alternative. It is often supposed that utilitarianism faces a serious problem if the future is infinitely long. For in that case, actions may produce an infinite amount of utility. And if that is so for most actions, then utilitarianism, it appears, loses most of its power to discriminate among actions. For, if most actions produce an infinite amount of utility, then few actions produce non-maximal utility, and so most actions are permissible.[1]

I will argue that potentially infinite futures create no major problems for utilitarianism. Utilitarianism has, I argue, the resources to distinguish among actions all of which produce infinite amounts of utility -- judging some permissible and some impermissible. For brevity of expression I will focus on act utilitarianism, but all the points apply equally well to many other traditional forms of utilitarianism.

## II. Situating the Problem

The above problem of infinite utilities arises only under certain conditions. In this section I will identify some of these conditions.

First, we should keep in mind, that although it is conceptually possible for the future to be infinitely long, it may not be physically possible. According to a widely accepted physical cosmology, the universe started with a big bang, has been expanding ever since, will at some time start contracting, and will eventually collapse into almost nothing. It is therefore quite plausible that the future -- at least the future that we can causally affect -- is only finite. So as a matter of empirical fact, there may be no problem for utilitarianism here. Still, it's at least a conceptual possibility that the future is infinitely long, so it's worth exploring how utilitarianism fares under such a scenario.

Second, an infinite future gives rise to infinite utilities only for certain sorts of utilitarian theories. Consider, for example, a <u>preference satisfaction</u> utilitarian theory according to which the utility produced by a given action in a given choice situation is determined by the extent to which its outcome satisfies the preferences of those <u>alive at the time of the choice</u>. Although people may care about the distant future (and of course they may not), the utility they assign to infinite futures may still be finite. If this is so (and this is almost always assumed for decision theory!), and there are only a finite number of people, then aggregate utility may be a finite sum (over people) of finite values.

The problem of infinite utilities arises (under certain conditions identified below) when the utilitarian principle is <u>temporally additive</u>, that is, such that the utility of a temporally extended state of affairs is the <u>sum</u> of the utility of each of its <u>temporal slices</u>. For such theories -- such as the classical total net pain over pleasure, and total happiness, principles -- each point in time has a certain associated utility, and the total utility is simply the sum of these utilities. The temporal summing over an infinitely long period of time is what gives rise to the possibility of utilities being infinite.

Not all utilitarian principles, we have just seen, are temporally additive. Consequently, infinite futures do not generate infinite utilities for all utilitarian principles in the manner under consideration (namely: by infinite sums). In what follows, then, we shall consider only temporally additive theories.

Finally, even for temporally additive theories, infinite futures do not automatically generate infinite utilities. If either of the following two conditions holds, then the total amount of utility produced by an action will still be finite: (1) there is a (finite) point in time in the future after which no utility is produced, or (2) utility is produced indefinitely into the future, but it asymptotically `dwindles away quickly enough' (see Diagram 1). In both such cases the total amount of utility produced over an infinite length of time is finite.[2]

INSERT DIAGRAM 1 HERE

Here and throughout the utility produced by an action at a time is to be understood as the <u>expected</u> (probability weighted) amount of <u>temporally discounted (if appropriate)</u> utility produced. The appeal to expected utility is necessary, since if indeterminism is true, there are several different amounts of utility that might be produced if a given action is performed. The appeal to temporally discounted utility is necessary so as to allow the possibility (but not the necessity) of discounting utility because of its temporal distance.

If utility is discounted for the mere passage of time, the most natural way is to weigh utility t years hence by a factor of $v^t$, for $0 < v < 1$. If this is done, then (discounted) utility will `dwindle away quickly enough', and so the total amount of utility produced will be finite. Traditional utilitarianism, however, does not use a temporal discount factor, and so does not avoid the problem in this manner.[3]

So the problem of infinite utilities being generated by infinite futures does not arise under all conditions, nor for all types of utilitarianism. It does arise, however, for temporally additive utilitarian theories when neither of the above two conditions is satisfied. In what follows, I shall argue that even under such conditions such theories have the resources to judge some actions with infinite utility permissible and other impermissible.

## III. A Revised Metric of Utility

Start by considering the case of Diagram 2 below. Here we have one action producing more utility at every point in time. Intuitively, most utilitarians would want to say that a1 produces more utility than a2. However, since both actions produce cardinally equal infinite amounts of utility, it is unclear how we could say this.[4] I will define a sense of `produces more' that enables us to say this.

DIAGRAM 2 GOES HERE

The root idea is this:

PMU: An action, a1 produces more utility than action, a2, if for each point in time a1 produces more utility at that time than a2.

This principle is to be understood as a partial stipulative definition of a technical notion that preserves the spirit of utilitarianism for cases of infinite utility. It distinguishes between two actions, both of which produce an infinite amount of utility, on the grounds that one action produces more utility at all points in time. My claim is that utilitarians will agree that the relevant notion of `produces more' for utilitarian theory entails PMU. If that is so, then in at least some cases utilitarianism judges that one action will `produce more utility' than a second, even when both produce an infinite -- and cardinally equal -- amount of utility. Consequently, utilitarianism can judge impermissible even actions that produce an infinite amount of utility.

Of course, we shouldn't expect very many pairs of actions to be such that one always produces more utility than the other. So, PMU doesn't take us very far. But there is a strengthening of PMU that takes us much further:

PMU*: An action, a1 produces more utility than action, a2, if and only if there is a time t such that for any later time t' the cumulative amount of utility produced by a1 up to t' is greater than that produced by action a2 up to t'.[5]

Under PMU* one action, a1, `produces more utility' than a second, a2, even if at some points in time the utility produced by a1 is lower than that produced by a2 -- if there is a time after which the cumulative utility produced by a1 is always greater than that produced by a2.[6] This is illustrated below in Diagram 3. Again, this is a stipulative definition intended to be useful to utilitarianism.

ADD DIAGRAM 3 HERE.

PMU* is a strengthening of PMU, since it agrees with PMU wherever PMU makes a distinction, but also makes distinctions that PMU does not make.

Although PMU* will treat some actions as `producing more utility' than a second action -- even if both produce an infinite amount of utility -- it also allows that two actions may be incomparable, i.e., such that neither produces at least as much utility as the other. This will be the case when, no matter how far out we go in time, the action that, in terms of the cumulative amount of utility produced, is tied or behind' at that point will at some later time `be ahead'. (See Diagram 4 below.) When such cases arise, there may be no action that is `maximal' in the sense of producing at least as much utility as any other action (since some actions will be incomparable). So it may look as if we have not yet avoided the problems with infinite utility (since it might appear that no action would be judged permissible).

DIAGRAM 4 GOES HERE

But there is a straightforward way of dealing with this problem, and that is to understand the utilitarian principle in the following manner:

U: An action is permissible if and only if no alternative produces more utility.

If all actions are comparable, then U and the more usual formulation (producing at least as much utility) are equivalent. If there is some incomparability, then the more usual formulation -- but not U -- may fail to yield judgements of permissibility.

For cases of temporally finite utility, PMU* (along with U) agrees exactly with the traditional approach of assessing what produces more utility. For cases of temporally infinite utility, however, it makes distinctions that the traditional approach does not make. It deems, for example, 100 utiles each day for eternity to be greater (in the sense relevant for utilitarian theory) than 10 utiles each day for eternity. And since such judgements are from the utilitarian perspective highly plausible, PMU* is more plausible than the traditional approach.

The key point is that for cases with infinite futures, utilitarians can, and should, appeal to the temporal distribution of the utilities-at-a-time in a way that is not legitimate for abstract integration (summation) problems. The traditional approach to utilitarianism is insensitive (after factoring out the probabilities of occurrence) to the temporal shifts of utility (e.g., 100 in 1990 and 50 in 1991 is viewed as equally good as 80 in 1990 and 70 in 1991). For cases of finite futures, PMU* -- like the more traditional approach -- entails this sort of neutrality to temporal shifting. For cases of infinite futures, however, PMU* is sensitive to the temporal shifting. For example, it judges <2,1,1 ....> as better than <1,1,1,....> even though the former is just a temporal shifting of the latter (shifting all utilities after the first one time earlier). So this is a departure from traditional utilitarian thought -- but it is a departure that utilitarians should willingly embrace. For it is a departure that allows utilitarianism to deal adequately with cases of temporally infinite utility.[7]

## IV. Conclusion

Although adopting PMU* and U preserves the spirit of utilitarianism, it may have some practical implications. For if there are many cases where <u>many</u> actions are undominated in the sense that no other action produces more utility, then there are many cases in which many actions are permissible. As a result, utilitarianism based on PMU* and U may be significantly less demanding than utilitarianism is usually thought to be. This is because PMU*'s metric for measuring utility and U's use of it are considerably more coarse-grained for the infinite case than for the finite case. Many utilitarians feel uncomfortable with the demanding nature of utilitarianism, and may well welcome this result. But welcome it or not, PMU* and U seem to be necessary and sufficient for preserving the coherence and spirit of utilitarianism in cases involving infinite utility.

In sum, the problem of infinite sums of utility (or value) stemming from infinite futures is an interesting and important problem for utilitarianism -- and many other sorts of goal-directed theories. This problem is largely defused, however, by allowing the theories to have metrics of utility (or value) that are appropriately -- as specified by PMU* -- sensitive to temporal distribution. Utilitarianism may have flaws, but total inability to deal with infinite futures is not one of them.[8]

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## Notes

1.Roughly this point has been made recently by M. Nelson, `Utilitarian Eschatology', American Philosophical Quarterly 28 (1991) pp.339-346.

2.Here I am simply reiterating a point made by Nelson. I thank John Broome for pointing out that in order for the sum of the utilities to be finite it is not enough for the utility values to `dwindle away' asymptotically with time. It must do so `quickly enough'. For example, if the utility at each time t is n/t for some fixed n, then the sum of such utility values will still be infinite. If, however, the utility at each time t is $n/t^2$ (which dwindles away more quickly), then the sum will be finite.

3.See, D. Parfit, Reasons and Persons (New York: Oxford University Press, 1984), Appendix F for a cogent argument against temporal discounting. Here I allow the possibility of discounting for the sake of generality, since not all temporal weightings ensure a finite sum. (A weighting of 1/t for utility t years hence, for example, does not.)

4.One solution that won't work is the following. From Cantor's work we know that not all infinite set are of the same size. The cardinality of the real numbers, for example, is greater than that of the natural numbers. So it is possible for one infinity to be greater than a second. In the present context, however, this won't be of much help. For, most of the cases will be ones in which all the options produce the same infinite amount of utility. If time is a continuum, for example, then there may be many actions that produce a continuum-many units of utility produced in such cases.

5.While this article was being refereed, John Broome pointed out to me that the problem of infinite utility streams had already been studied by economists. I was both pleased and disappointed to learn that my proposed PMU* had already been proposed, and apparently endorsed, by economists (they call it `the overtaking criterion'). I was pleased that my proposal had independent backing, but disappointed that I wasn't the first to make the proposal! See, for example, T.C. Koopmans, `Representation of

Preference Orderings Over Time', Ch.4 of Decision and Organization, edited by C.B. McGuire and R. Radner (Amsterdam: North Holland Publishing Company, 1972).

6.PMU* is naturally complemented by the following: An action, a1 produces the same amount of utility as action, a2, if and only if there is a time t such that for any later time t' the cumulative amount of utility produced by a1 up to t' is equal to that produced by action a2 up to t'.

7.Note that like traditional utilitarianism, PMU* is temporally neutral when two or more temporal utilities are permuted (i.e., exchanged or switched). This is not true of the following (Pareto-like) stronger principle (which one might erroneously consider adopting):

PMU**: An action, a1 produces more utility than action, a2, if and only if there is a time t such that (1) for each later time t' the cumulative amount of utility produced by a1 up to t' is at least as great as that produced by action a2 up to t', and (2) for some later time t' the cumulative amount of utility produced by a1 up to t' is greater than that produced by action a2 up to t'.

To judge an action as producing more utility than a second, this principle, unlike PMU*, does not require that the action always be ahead after some point in time with respect to the cumulative utility produced. It only requires that after some point it never be behind, and that it be ahead at least sometimes (and perhaps tieing at the rest). As a result, PMU** differs from the traditional approach even in the finite case. For it would judge 2 followed 1 utiles and then nothing else as greater than 1 followed by 2 and then nothing else. So PMU**, unlike PMU*, is a radical departure from the traditional approach. I thank Charles Dresser for making this clear to me.

8.Thanks to John Broome, Charles Dresser, Tony Ellis, Bob Frazier, Brad Hooker, Mark Nelson, Graham Oddie and Teddie Seidenfeld for helpful comments.