PETER VALLENTYNE

HOW TO COMBINE PARETO OPTIMALITY
WITH LIBERTY CONSIDERATIONS

ABSTRACT. I argue that the liberty condition of Sen's important impossibility of a Paretian liberal result is not a condition that liberals (or libertarians) would accept. The problem is that an appropriate liberty condition must be formulated in terms of consent – not in terms of preference. To formulate an adequate condition the framework needs to expand from collective choice rules (which only take information about preferences as input) to rights-based social choice rules (which also take as input information about which options have been consented to and which would violate someone's rights). I formulate a more adequate liberty condition based on the notion of consent that is acceptable to liberals, and then show that Pareto optimality is incompatible even with that condition. I then show how the liberty condition can be weakened in a plausible manner, and describe an interesting class of theories – rights-based Paretian theories – that satisfy the Pareto optimality requirement while being sensitive to liberty considerations.

Keywords: Sen, Pareto optimality, liberty, rights, liberal.

I. INTRODUCTION

A central result in the theory of social choice is Sen's impossibility of a Paretian liberal theorem. According to this result there is no social choice rule defined for all preference profiles that satisfies both a Pareto optimality condition and a weak liberty condition. More precisely, the result is the following.

Let X be the set of all *alternatives* over which individuals have preferences. It is assumed that there are at least three alternatives. Let I be the set of *individuals in society*, and let n be the number of such individuals. It is assumed that $n \geqslant 2$. Let R_i be individual i's *weak preference relation*, and P_i be his/her strict preference relation. It is assumed that for each individual i in I R_i is reflexive, transitive and complete. A *preference (utility) profile* is an n-tuple of weak preference relations (utility functions), one for each member of society. Let O be

the set of all *option sets*, i.e., the set of all non-empty subsets of X. A *choice function* (in this context) is a function that for any given option set specifies a non-empty subset. An ordinal (cardinal) *social choice rule* is a function that takes as input preference (utility) profiles, and perhaps other things (e.g., information about rights), and specifies a choice function (over option sets). An ordinal (cardinal) *collective choice rule* is a particular kind of social choice rule, it takes as input *only* preference (utility) profiles. Sen formulated his result only in terms of ordinal collective choice rules, but the result is valid for all social choice rules.

Consider, then, the following conditions, where f is an arbitrary social choice rule:

> U (Universal Preference Domain): The social choice rule, f, is defined for all logically possible preference (utility) profiles.
>
> P (Weak Pareto Optimality): For any preference (utility) profile, the choice function that f selects is such that: if x and y are options, and everyone strictly prefers x to y, then y is not in the choice set.
>
> L (Liberty Condition): For each individual there are at least two alternatives such that the choice function selected by f for any given preference (utility) profile is such that: if both these two alternatives are options and the individual strictly prefers one to the other, then the less preferred option is not in the choice set.

Sen's theorem is:

> THEOREM. *The Impossibility of a Paretian Liberal*: There is no social choice rule satisfying conditions U, L, and P.[1]

The validity of the theorem is uncontroversial. The significance of the theorem for ethical and political theory depends on the plausibility of the three conditions. If the conditions that Sen imposes are implausibly strong, weakening them may yield a possibility result. There are three possibilities:

(1) One might argue that Condition U needs to be weakened, but I accept this condition and shall say nothing about it.

(2) One might argue that Condition P needs to be weakened. I shall discuss one weakening of P, but ultimately I shall accept P.

(3) Finally, one might argue that Condition L needs to be weakened. I shall argue that L does not adequately capture even a minimal aspect of the liberty condition that liberals and libertarians would defend. In order to adequately deal with liberty considerations social choice rules need to take as input (a) information about which alternatives would *violate a person's rights* if he/she has not consented to that option, and (b) information about which alternatives people have *consented to*. Based on this input I shall formulate a new, more adequate liberty condition, and show that an impossibility result holds even for this more adequate liberty condition. After showing how this new condition can be combined with a restricted version of the Pareto condition, I argue that, although the new liberty condition is acceptable to libertarians, it is unacceptably strong. I formulate a weaker liberty condition (still acceptable – as far as it goes – to libertarians), and show that a specific social choice rule satisfies it, and U and P.

My goal in this paper is not merely to show how the conditions on which Sen's result rest can be weakened so as to avoid the impossibility result. It is also to defend a class social choice rules called 'rights based Paretian rules', as being highly attractive ways of combining liberty considerations with Pareto optimality.

Before getting down to business one methodological remark is in order.

2. ON THE INTERPRETATION OF SOCIAL CHOICE THEORY

In assessing the plausibility of the conditions of a social choice theorem it is extremely important to distinguish between different interpretations of the set of alternatives and the functions defined over them. Conditions that are plausible on one interpretation may not be plausible on another. Throughout I shall be concerned with the *ethical theory interpretation*, according to which: (1) X (the set of alternatives)

is the set of conceptually possible actions of some individual in some choice situation; (2) an option set (a member of O) is a set of all and only those actions that under some conceptually possible scenario are feasible for the individual in the choice situation; and (3) a social choice rule is an ethical theory that, for a given preference (utility) profile – and perhaps other inputs – specifies a choice function, cf, such that for a given option set, S, all and only those actions judged *permissible* are members of cf(S).

Note that on the ethical theory interpretation (and the political interpretation below) there is no assumption that the elements of S that are judged *permissible* are also judged as *equally good* from a social viewpoint. Some may be judged as better than others (for example, if they are judged permissible as long as they are above some minimum level of goodness), or there may be no social ranking of alternatives at all.

The ethical theory interpretation can be contrasted both with the political theory interpretation and with the practical choice mechanism interpretation. On the *political theory interpretation* the set of alternatives is – not the set of actions, but rather – the set of basic social structures (basic ways of organizing society, e.g., socio-politico-economic constitutions), and the social choice rules are political theories that determine for any set of feasible basic social structures which are morally permissible. The criteria of adequacy (for example, concerning liberty considerations) for political theories may be significantly different from those for ethical theories.

On the *practical choice mechanism interpretation* the distinguishing feature is not the set of alternatives (which may be courses of action, basic social structures, or whatever) but rather the fact that the social choice rule is interpreted as a mechanism actually to be instituted in society as a means for making social decisions (e.g., electing leaders, or selecting a constitution). On this interpretation it is not people's true preference that serve as input, but rather what they register their preferences to be (e.g. by means of voting). Consequently two important considerations are: (1) the extent to which people are *able* to reliably report the required information about their preferences (utilitarianism which requires information about cardinal utility would be too demanding in this regard); and (2) the extent to which people

will be *motivated* to honestly report the required information about their preferences (voting schemes may encourage people to misrepresent their preferences so as to manipulate the voting results to their advantage).[2] Neither of these considerations are relevant to the ethical theory interpretation.

In what follows, then, I shall be concerned only with the ethical theory interpretation, i.e., with the interpretation of social choice theory as a metatheory of ethical theories.

3. THE IMPOSSIBILITY OF A PARETIAN LIBERAL: A NEW VERSION

In this section I shall argue that Sen's Condition L does not adequately capture even a minimal aspect of the spirit of libertarianism in that libertarians would reject the condition. I shall formulate a more acceptable liberty condition, and show that the impossibility of a Paretian liberal result still holds with the new liberty condition.

To see the inadequacy of L, consider *libertarianism*. According to this ethical theory there are certain features of actions – such as being a physical harming, a lying, or a breaking of a promise or agreement – such that any action that has one of those features is forbidden except if the relevant persons have consented to such actions.[3] The basic idea is that there are certain rights-generated constraints – call them *the libertarian constraints* – on how individuals may be treated without their consent. Libertarianism judges an action permissible just in case it violates none of the libertarian constraints without the consent of the protected persons.

Libertarianism is representable as a social choice rule in the following manner. Let a *quasi-choice function* be a function such that for any given option set specifies some (possibly empty) subset. (Quasi-choice functions are exactly like choice functions, except that the subset they specify may be empty.) Let 'con$_i$' designate individual i's *consent function*, i.e., a quasi-choice function such that for any option set, S, con$_i(S)$ is the set of alternatives in S to which individual i has consented; and understand a *consent profile* (con^n) to be an n-tuple of consent functions, one for each individual. Let 'cv$_i$' (for 'constraint violation') designate individual i's *constraint function*, i.e., a quasi-

222   PETER VALLENTYNE

choice function such that for any option set S $cv_i(S)$ is the set of alternatives in S that, if chosen without i's consent, would violate i's rights (as specified by some theory); and understand a *constraint profile* (cv^n) to be an n-tuple of constraint functions, one for each individual.

Along with preference profiles, consent and constraint profiles serve to individuate choice situations. Consent profiles specify for each option set which options have been consented to, and constraint profiles specify for each option set which options violate the relevant (as specified by some moral theory) constraints.[4]

Libertarianism is representable as a social choice rule – call it lib – that for any given preference profile, R^n, any given consent profile, con^n, and any given libertarian constraint profile, cv^n, specifies the choice function that for a given option set specifies all and only those options that violate none of the libertarian constraints without the consent of the affected individuals.[5] In symbols:

> *Libertarianism* (lib): For any given preference profile, R^n, any given consent profile, con^n, and any given libertarian constraint profile, cv^n: $lib(R^n, con^n, cv^n)$ = the choice function, cf, such that for any given option set, S, $cf(S)$ = $\{x \in S: {\sim}(\exists i)\ (x \in cv_i(S)\ \&\ x \notin con_i(S))\}$.

Libertarianism is *not* representable as a collective choice rule. This is because collective choice rules take as input only preference (or utility) profiles; they do not take as input information about which of some specified constraints are violated or about which options have been consented to. Let us call social choice rules that take preference (utility), consent, and constraint profiles (of a specified kind) and specify choice functions *rights-based social choice rules* (hereafter: RBSCRs). For simplicity I shall focus on ordinal RBSCRs, but the results of this paper apply equally well to cardinal RBSCRs.

Collective choice rules are a special kind of RBSCR: they ignore the consent and constraint profile input and specify choice functions solely on the basis of the preference profile input. The libertarian social choice rule, lib, is a RBSCR, but not a collective choice rule. Of course, as an RBSCR, lib formally takes preferences profiles as input,

but it totally ignores them. It's output depends only on the consent and
constraint profiles. Consequently, lib does not satisfy L, since lib
ignores the preference profiles, and L requires a certain sensitivity to
that input.

We have then:

THEOREM 1. *Lib does not satisfy L.*

Proofs of all theorems (as well as symbolic statements of all conditions)
are given in the Appendix.

Condition L is a plausible liberty condition only if one assumes that
the selection of a less preferred option of an individual's privileged pair
(when both are available) is a violation of that person's rights. But
according to libertarianism no rights are violated when *consent* has
been given to the less preferred alternative. Libertarians agree that
(under normal conditions) the permissibility of (for example) my
parting my hair on the left depends on my views. They deny, however,
whereas L asserts, that it is wrong for me to part my hair on the left
when I prefer to part it on the right. It's up to my volition (as
manifested by my consent) – and not my preferences – whether parting
my hair on the left is permissible.[6]

Condition L is not an adequate liberty condition because it does not
recognize the role of consent in liberty. A more appropriate liberty
condition would be based on the notions of rights-generated con-
straints and of consent. Below I shall formulate such a condition for
RBSCRs.

First, however, we should note that different RBSCRs will (in
general) be based on different conceptions of rights (e.g., libertarian
rights vs. welfare rights). Thus, they will recognize different constraint
profiles as relevant (e.g., libertarian constraint profiles vs. (some sort
of) 'welfare' constraint profiles). Let us say that different RBSCRs
have different criteria concerning what constraint profiles are *admis-
sible*. Here we shall not be concerned with substantive criteria of
admissibility, since these will vary from RBSCR to RBSCR. We shall,
however, be concerned with one formal criterion of admissibility. A
minimal liberty condition for a RBSCR is that an admissible constraint
profile must be non-empty – in the sense that the constraints must rule

224   PETER VALLENTYNE

out at least some options (otherwise the constraints are empty).
Somewhat more strongly, a minimal liberty condition on the admis-
sibility of constraint profiles is the following:

> NEC* (Non-Empty Constraints): A constraint profile is
> admissible only if it is such that for each individual there is
> some option of some option set that violates the constraints
> protecting him/her.

The libertarian constraints clearly satisfy NEC*, since, for example
almost any option set that includes the option of the pointless torturing
of a person, violates the libertarian constraints protecting that person.

The satisfaction of NEC* ensures that the rights-generated con-
straints recognized by the RBSCR are not empty. It does not ensure,
however, that the RBSCR judges *impermissible* options that violate
the relevant constraints without appropriate consent. For NEC* does
not require any connection between permissibility and the satisfaction
of the constraints. The next conditions require that permissibility be
sensitive to whether the constraints are satisfied.

> SCPL* (Strong Consent Protected Liberty Condition): For
> all admissible constraint profiles, all preference and consent
> profiles, all individuals, and all options of all option sets: if
> the option violates the constraints on how the individual
> may be affected, and the individual does not consent to that
> option, then it is not judged permissible.

It is easy to see that lib satisfies SCPL*. Any option that violates a
libertarian constraint is judged impermissible, if the relevant individual
does not consent to it – no matter what people's preferences are. (For
example, Smith's punching of Jones in the face is judged impermiss-
ible, if Jones has not consented to it.)

SCPL* requires that *every* option that violates the constraints of *any*
affected person without his/her consent be judged impermissible. A
weaker requirement (entailed by SCPL* in the presence of NEC*) is
the following:

WCPL* (Weak Consent Protected Liberty Condition): For all admissible constraint profiles, there is at least one individual and at least one option of at least one option set such that for all consent profiles, and all preference profiles: if the individual does not consent to the option (for that option set), then the option is judged impermissible.

WCPL* and SCPL* require that there be options for which an individual's consent is *necessary* for the option to be judged permissible. Libertarians would also want to impose a condition that requires that at least sometimes an individual's consent is *sufficient* for an option to be judged permissible. One such condition is:

SCJL* (Strong Consent Justifying Liberty Condition): For all admissible constraint profiles, and all individuals, there is at least one option of at least one option set such that for all consent profiles, and all preference profiles: if the individual consents to the option (for the option set), then the option is judged permissible.

A weaker condition – entailed by SCJL* – would require only that the condition hold for at least one person:

WCJL (Weak Consent Justifying Liberty Condition): For all admissible constraint profiles, there is an individual, and at least one option of at least one option set such that for all consent profiles, and all preference profiles: if the individual consents to the option (for the option set), then the option is judged permissible.

Lib satisfies both WCJL* and SCJL*. For example, lib holds that (under normal conditions) it is permissibe for the barber to cut a person's hair if he/she consents to it.

Lib also satisfies U*, the following universal domain condition for RBSCRs:

226   PETER VALLENTYNE

U* (Universal Domain for RBSCRs): The RBSCR is de-
fined for all logically possible combinations of preference,
consent, and admissible constraint profiles.

Lib, does not, however, satisfy the following Pareto condition for
RBSCRs:

SP* (Strong Pareto for RBSCRs): For all admissible con-
straint profiles, and all preference profiles, consent profiles,
option sets, and individuals: if x and y are options, everyone
weakly prefers x to y, and someone strictly prefers x to y,
then y is not judged permissible.

Nor does it satisfy the following weaker Pareto condition:

P* (Weak Pareto for RBSCRs): For all admissible con-
straint profiles, and all preference profiles, consent profiles,
option sets, and individuals: if x and y are options, and
everyone strictly prefers x to y, then y is not judged
permissible.

Libertarianism satisfies neither of these conditions because it is not
directly sensitive to what preferences people have. (It is, of course,
indirectly sensitive to their preferences, since in general people consent
to things that they desire.) For even if everyone prefers a to b, b is
judged permissible if it violates none of the constraints.

Thus, we have established:

THEOREM 2. *Lib satisfies* U^*, NEC^*, $SCPL^*$ (*and* $WCPL^*$),
$SCJL^*$ (*and* $WCJL^*$), *but not* P^* (*or* SP^*).

Indeed, roughly this same reasoning establishes the following
stronger results (proofs are given in the Appendix):

THEOREM 3. *No RBSCR satisfies* U^*, $WCPL^*$ (*or* $SCPL^*$ *and*
NEC^*), *and* P^* (*or* SP^*).

THEOREM 4. *No RBSCR satisfies U*, WCJL* (or SCJL*), and P* (or SP*).*

The validity of these two theorems rests on the basic conflict between the role given to preferences by P*, and the role given to consent by WCPL* and by WCJL*. Under certain circumstances the only options that violate no one's rights are Pareto sub-optimal. Consequently, under such circumstances no option will satisfy both P* and WCPL*. Likewise, Pareto optimality considerations can rule out any option, and so if P* is accepted, consent cannot be sufficient for permissibility (as required by WCJL*).

Thus, reformulating the liberty condition in terms of consent does not avoid the impossibility result. No social choice rule can satisfy a plausible liberty condition and P*. Sen's orginal liberty condition was not plausible, but the problem he uncovered remains.

Something must give way. Either the liberty condition must be weakened, or the Pareto condition must be weakened. I shall now explore, but only to reject, a way of avoiding the impossibility by weakening P*. In the section after next I shall defend a way of avoiding the impossibility result by weakening WCPL*.

4. WEAKENING THE PARETO CONDITION: RIGHTS CONSTRAINED PARETIANISM

Following the usual terminology, let us say: (1) that an option is *Pareto optimal relative to a set S* (of which it is a member) just in case there is no member of S that everyone weakly prefers, and someone strictly prefers, to it; and (2) that an option *Pareto dominates* another just in case it is weakly preferred by all and strictly preferred by some.

For symbolic brevity let 'po(S, R^n)' designate the set of Pareto optimal elements in S relative to the preference profile R^n. In symbols:

$$\text{po}(S, R^n) = \{x \in S : {}^\sim(\exists y)[y \in S \,\&\, (\forall i)Riyx \,\&\, (\exists i)(Piyx)]\}$$

One way of combining rights considerations with Pareto optimality

considerations is to make the rights considerations *prior* to Pareto
optimality considerations. Such an approach requires Pareto optimality
relative to the set of options that violate no one's rights. Let us
consider, then, *libertarian constrained Paretianism*, lcp for short, which
judges an action permissible just in case, of those actions that violate
none of the libertarian constraints except by consent, it is Pareto
optimal.[7] In symbols:

> Libertarian Constrained Paretianism (lcp): For any given
> preference profile, R^n, consent profile, con^n, and libertarian
> constraint profile, cv^n: $lcp(R^n, con^n, cv^n) =$ the choice func-
> tion, cf, such that for all option sets, S, $cf(S) = po(cf^*(S),$
> $R^n)$; where $cf^* = lib(R^n, con^n, cv^n)$.

Libertarian constrained Paretianism makes the libertarian con-
straints prior to Pareto optimality considerations. Like libertarianism it
prohibits the violation of the libertarian constraints, and so satisfies
SCPL* (and WCPL* and NEC*). Unlike libertarianism it further
prohibits actions that are not Pareto optimal relative to the set of
actions that satisfy the constraints.

Libertarian constrained Paretianism does not satisfy P* (or SP*).
For although it judges permissible only options that are Pareto optimal
relative to the set of alternatives that satisfy the libertarian conditions, it
does not require such options to be Pareto optimal *relative to the set of
all options*. An option that it judges permissible may be Pareto
dominated by some other option that is not judged permissible (be-
cause it violates someone's rights).

Although libertarian constrained Paretianism does not satisfy P* (or
SP*), it does satisfy the following weaker condition:

> RP* (Restricted Weak Pareto): For all admissible constraint
> profiles, and all preference profiles, consent profiles, and
> option sets: if x is judged permissible and is strictly pre-
> ferred by all to y, then y is not judged permissible.

Libertarian constrained Paretianism satisfies even the strong form of
restricted Pareto condition:

RSP* (Restricted Strong Pareto): For all admissible constraint profiles, and all preference profiles, consent profiles, and option sets: if x is judged permissible and Pareto dominates y, then y is not judged permissible.

Unlike their unrestricted counterparts, these conditions require that an alternative be rejected if it is less preferred by all to some other alternative *only if the more preferred alternative is in the choice set.*[8] Libertarian constrained Paretianism satisfies both RP* and RSP*, because it requires Pareto optimality relative to the set of options that satisfy the constraints.

Because libertarian constrained Paretianism also satisfies U* (is defined for all logically possible combinations of preference, consent, and admissible constraint profiles) and NEC*, we have the following result:

THEOREM 5. *Libertarian constrained Paretianism satisfies U*, NEC*, SCPL* (and WCPL*), and RSP* (and RP*), but not P* (or SP*).*

So, liberty considerations (of the rights protected variety) can be combined with Pareto considerations, provided that nothing stronger than the restricted Pareto conditions is imposed.

Hard core libertarians will, of course, reject even the above restricted versions of the Pareto conditions on the grounds that satisfying the libertarian conditions (i.e., not violating any constraint without the protected person's consent) is not only a necessary condition for being permissible, but also a sufficient condition. Any restriction beyond that against constraint violation (for example requiring Pareto optimality relative to the options that satisfy the constraints) is, they would claim, an illegitimate restriction on the liberty of the agent. More specifically, libertarians would object that libertarian constrained Paretianism does not satisfy WCJL* (Weak Consent Justifying Liberty Condition), since for no option is the consent of any given agent sufficient for that option to be judged permissible. To be judged permissible by libertarian constrained Paretianism the option must be Pareto optimal, and that depends on the preferences of others.

230   PETER VALLENTYNE

So, hard core libertarians will reject libertarian constrained Paretianism, because they reject even the restricted Pareto conditions. Nonetheless, those who find merit in WCPL* (consent protected liberty) and RSP* (restricted Pareto optimality) may be attracted to libertarian constrained Paretianism.

The theory has a serious defect, however. As mentioned above, it does not satisfy P*. To illustrate this, consider the following choice situation:

Option	Satisfy Libertarian Conditions?	Preference Ranking	
		1	2
x1	yes	3rd	3rd
x2	yes	2nd	2nd
x3	no	1st	1st

Here relative to the set of options that satisfy the libertarian conditions (i.e. $\{x1, x2\}$) only $x2$ is Pareto optimal. Consequently, libertarian constrained Paretianism judged only $x2$ permissible. It judges $x3$ impermissible (since it violates someone's rights) – even though everyone prefers it to $x2$.[9] But if $x2$ is judged permissible, and everyone prefers $x3$ to $x2$, why isn't $x3$ also judged permissible? To judge it impermissible simply because it violates the constraints without consent is taking the constraints too seriously. The constraints should be taken seriously only to the extent that they protect human welfare. Pareto optimality, that is, should not take second place to the libertarian constraints.

A main problem with libertarian constrained Paretianism, then, is that it does not satisfy the following monotonicity condition:

> SM* (Strong Monotonicity): For all admissible constraint profiles, and all preference profiles, consent profiles, and option sets: if x and y are options, x is judged permissible, everyone weakly prefers y to x, and some strictly prefer y to x, then y is also judged permissible.

Nor does it satisfy the following weaker monotonicity condition:

M* (Weak Monotonicity): For all admissible constraint profiles, and all preference profiles, consent profiles, and option sets: if x and y are options, x is judged permissible, and everyone prefers y to x, then y is also judged permissible.

These two conditions require that permissibility be positively sensitive to welfare. If one option is permissible, and another option is Pareto superior to it, then the second option should also be permissible.

It is easy to show that the conjunction of (strong) monotonicity with the restricted (strong) Pareto optimality condition is equivalent to the unrestricted (strong) Pareto condition. Thus, libertarian constrained Paretianism satisfies the restricted, but not the monotonicity part of the unrestricted Pareto conditions. Because the monotonicity conditions are highly plausible, libertarian constrained Paretianism does not represent an adequate solution to the problem of how to combine Pareto considerations with liberty considerations.

I shall now present a theory similar in spirit, that satisfies the unrestricted Pareto conditions, but satisfies only a weakened version of WCPL*. This theory will illustrate a way in which liberty considerations can be combined with Pareto optimality, provided one weakens the liberty condition in a certain plausible manner.

5. WEAKENING THE LIBERTY CONDITION: RIGHTS BASED PARETIANISM

For brevity, let us call the actions judged permissible by libertarian constrained Paretianism *the LCP candidates*. Let us now consider a theory – call it *libertarian-based Paretianism* – that judges an action permissible just in case it is Pareto optimal and each person weakly prefers it to some LCP candidate. The new theory then is:

Libertarian-based Paretianism (lbp): For any given preference profile, R^n, consent profile, con^n, and libertarian constraint profile, cv^n: lbp(R^n, con^n, cv^n) = the choice function, cf, such that for all S, cf(S) = {$x \in S$: [$x \in$ po(S, R^n)] &

$$[(\forall i)(\exists y) \ (y \in cf^*(S) \ \& \ Rixy)], \ \text{where}$$
$$cf^* = lcp(R^n, con^n, cv^n).$$

Unlike libertarian constrained Paretianism, libertarian-based
Paretianism satisfies SP* and P*. This is because, unlike the former, it
takes Pareto optimality relative to the option set (not merely relative
to the options that satisfy the libertarian conditions) as a necessary
condition for permissibility. To be judged permissible, an action must
be both Pareto optimal and weakly preferred by each person to some
LCP candidate. Consequently, both SP* and P* are satisfied.

Unlike libertarianism and libertarian constrained Paretianism, liber-
tarian-based Paretianism does not satisfy WCPL*. This is because it
does not treat the libertarian constraints as real constraints, but only as
reference point determiners. Even options that violate the constraints
without the relevant consent are judged permissible, if they are Pareto
optimal and weakly preferred by each person to some LCP candidate.

Although libertarian-based Paretianism does not satisfy WCPL*, it
is sensitive to whether the libertarian constraints are satisfied. For,
although the constraints may be violated, they nonetheless determine
(via the determination of the LCP candidates) a minimum level of
preference satisfaction for each person. No action which someone
prefers less than *all* LCP candidates is judged permissible. Libertarian-
based Paretianism thus satisfies the following condition:

> RPLP* (Reference Point Liberty Protection Condition):
> For each admissible constraint profile, each consent profile,
> each preference profile, each individual, and each option of
> each option set: if the individual disprefers the option to *all*
> other options in the option set that satisfy the constraints
> (i.e., that do not violate the constraints without the relevant
> consent), then that option is not judged permissible.

RPLP* is entailed by SCPL*, but not by WCPL* (since RPLP*
requires the constraints to be relevant for all option sets, and WCPL*
only requires that they be relevant for some option set). Unlike SCPL*
and WCPL*, RPLP* does not require that any option that violates the
libertarian conditions be judged impermissible. Such options may be

judged permissible if they are weakly preferred by each person to
some option that satisfies the libertarian conditions. Consequently,
although SCPL* and WCPL* are incompatible with P*, RPLP* is not.

Thus, given that libertarian-based Paretianism also satisfies U* and
NEC*, we have the following result:

THEOREM 6. *There is an RBSCR – e.g., libertarian-based Paretianism – that satisfies U*, NEC*, SP* (and P*), and RPLP*.*

Thus, while there is no RBSCR that satisfies U*, P*, and WCPL*
(Theorem 3), there is a RBSCR (e.g., libertarian constrained Paretianism) that satisfies U*, NEC*, RSP* (and RP*), and SCPL* (and
WCPL*) (Theorem 5), and there is a RBSCR (e.g., libertarian based
Paretianism) that satisfies U*, NEC*, SP* (and P*), and RPLP*
(Theorem 6). Given the choice (posed by the three theorems taken
together) between giving up P* and giving up WCPL*, many will find
it more plausible to give up WCPL*. Theorem 6 establishes that in
giving up WCPL*, one need not give up all liberty considerations. The
rights-generated constraints can still be relevant for determining a
minimum welfare entitlement for each person.

Libertarian-based Paretianism does not, however, satisfy WCJL*.
WCJL* requires that there be at least one individual and at least one
option of at least one option set, such that individual's consent is
sufficient for the permissibility of that option – no matter what people's
preferences are. Theorem 4 established that WCJL* and P* conflict in
the presence of U*. If Pareto optimality is required for permissibility,
consent cannot be sufficient.

CONCLUSIONS

My reaction to Sen's impossibility of a Paretian liberal result, then, is:
(1) to broaden the framework from collective choice rules to RBSCRs
so that the social choice rules may take as input information about
constraint violation and consent; and (2) to reject Sen's L, because it
connects permissible choice only to the *preferences* of individuals,
whereas the sort of liberty condition on which liberals and libertarians
would insist connects permissible choice with *rights* and *consent*. This

shows that Sen's result does not establish that Paretian liberalism is impossible.[10]

Nonetheless, there is a problem, for there are plausible consent-based liberty conditions which are incompatible with Pareto optimality. WCPL* requires that there be at least one individual, and at least one option of some option set for which that person's consent is *necessary* for the permissibility of the option; and WCJL* requires that there be at least one individual, and at least one option of some option set for which that person's consent is *sufficient* for the permissibility of the option. Both of these conditions would be accepted by almost all liberals, but are incompatible with P*. Liberals are therefore forced to choose between these liberty conditions and Pareto optimality.

I have presumed – rather than argued – that Pareto optimality is a very plausible condition, and showed how WCPL* can be weakened so as to be compatible with Pareto optimality. The weakened condition, RPLP*, requires that for each individual there is some option of some option set for which that person's consent is *necessary* for the permissibility of the option *if that person disprefers that option to all options that do not violate anyone's rights*. This condition requires that at least one person have *conditional* veto power for at least one option. Those, such as libertarians, who view rights as protecting the *control* that agents have over their lives will find RPLP* grossly inadequate as a characterization of the role of rights considerations. Those who view rights as protecting the *welfare* of agents, however, will find RPLP* much more plausible. For, unlike the stronger liberty conditions, it is compatible with Pareto optimality. In particular, libertarian-based Paretianism satisfies both RPLP* and P*.

For concreteness I have focused on libertarian-based Paretianism, but everything said above about libertarian-based Paretianism applies equally well to rights-based Paretian theories generally. *Rights-based Paretian* theories are theories that judge an option permissible if and ony if (1) it is Pareto optimal and (2) each individual weakly prefers it to some option that (a) violates none of the constraints generated by certain specified rights without the relevant consent, and (b) is Pareto optimal relative to the options that satisfy condition (a). Libertarian-based Paretianism is a rights-based Paretian theory that is based on the libertarian constraints. Other rights-based Paretian theories are based on the constraints generated by other conceptions of rights.[11]

Rights-based Paretian theories combine a number of attractive features of pure rights theories (such as libertarianism) and utilitarianism without having their defects. Like pure rights theories (and rights-constrained Paretian theories), but unlike utilitarianism, they are sensitive to rights-generated constraints, require neither cardinal utility information nor interpersonal comparisons of utility, and can leave the agent a significant amount of liberty. Like utilitarianism, but unlike pure rights theories (and rights constrained Paretian theories), rights-based Paretian theories require options to be Pareto optimal in order to be judged permissible.

The plausibility of a rights-based Paretian theory will depend on crucially on the rights that it recognizes. I have not undertaken here to defend any particular conception of rights. My claim is only that rights considerations of some sort are relevant, and that rights-based Paretian theories recognize rights in a plausible manner. In particular, because rights-based Paretian theories satisfy U*, SP* and RPLP*, rights-based Paretian theories are a plausible way of combining Pareto optimality and liberty considerations.[12]

APPENDIX

I state here in symbols the definitions and conditions given verbally in the text, and provide proofs of the theorems.

Libertarianism (*lib*): For all R^n, con^n, and admissible cv^n: $lib(R^n, con^n, cv^n) =$ the choice function, cf, such that for all S, $cf(S) = \{x \in S: \sim(\exists i) (x \in cv_i(S) \ \& \ x \notin con_i(S))\}$.

L^* (*Liberty*): For all i there are at least two distinct alternatives, x and y, such that for all R^n, con^n, cv^n, and S: (1) if x, $y \in S$ and Pixy, then $y \notin cf(S)$, and (2) if x, $y \in S$ and Piyx, then $x \notin cf(S)$; where $cf = f(R^n, con^n, cv^n)$, and f is an RBSCR.

LEMMA 1. *Lib does not satisfy L^*.*

Proof. Suppose for a reductio that lib does satisfy L^*, with individual 1 having control over a and b. Let S be an arbitrary option set containing a and b; let R^n be arbitrary preference profile for which 1 prefers a to b (i.e., P$1ab$); let con^n be an arbitrary consent profile for

236   PETER VALLENTYNE

which 1 has consented to b (in S) (i.e., $b \in \text{con}_1(S)$); and let cv^n by an arbitrary libertarian constraint profile for which b violates none of the constraint on how others may be treated in S. Let $\text{cf} = \text{lib}(R^n, \text{con}^n, \text{cv}^n)$. Then by L^*, $b \not\subseteq \text{cf}(S)$ (since Plab). But by the definition of lib, and the fact that b has 1's consent and violates no constraints on how others may be treated, $b \in \text{cf}(S)$, which yields a contradiction. So lib does not satisfy L^*.

THEOREM 1. *Lib does not satisfy L.*
    Proof. Trivial, since L^* is simply a specification of L for RBSCRs.

NEC* (*Non-Empty Constraints*): A constraint profile, cv^n, is admissible only if for all i there is an x in some S such that $x \in \text{cv}^i(S)$.

SCPL* (*Strong Consent Protected Liberty*): For all admissible cv^n, all R^n, all i, all x, all S, and all con^n; if $x \in \text{cv}_i(S)$, and $x \not\subseteq \text{con}_i(S)$, then $x \not\subseteq \text{cf}(S)$; where $\text{cf} = f(R^n, \text{con}^n, \text{cv}^n)$, and f is an RBSCR.

WCPL* (*Weak Consent Protected Liberty Condition*): For all admissible cv^n: there is an i, an S, and an $x \in S$ such that for all R^n, and all con^n: if $x \not\subseteq \text{con}_i(S)$, then $x \not\subseteq \text{cf}(S)$; where $\text{cf} = f(R^n, \text{con}^n, \text{cv}^n)$, and f is an RBSCR.

SCJL* (*Strong Consent Justifying Liberty Condition*): For all admissible cv^n, all i: there is an S, and an $x \in S$ such that for all con^n, and all R^n: if $x \in \text{con}_i(S)$, then $x \in \text{cf}(S)$; where $\text{cf} = f(R^n, \text{con}^n, \text{cv}^n)$, and f is an RBSCR.

WCJL* (Weak Consent Justifying Liberty Condition): For all admissible cv^n, there is an i, an S, and an $x \in S$ such that for all con^n, and all R^n: if $x \in \text{con}_i(S)$, then $x \in \text{cf}(S)$; where $\text{cf} = f(R^n, \text{con}^n, \text{cv}^n)$, and f is an RBSCR.

U* (*Universal Domain for RBSCRs*): The RBSCR is defined for all logically possible combinations of preference, consent, and admissible constraint profiles.

SP* (*Strong Pareto for RBSCRs*): For all R^n, con^n, admissible cv^n, S,

and x, y: if $x, y \in S$, $(\forall i)$ Rixy, and $(\exists i)$ Pixy, then $y \notin \mathrm{cf}(S)$; where $\mathrm{cf} = f(R^n, \mathrm{con}^n, \mathrm{cv}^n)$, and f is an RBSCR.

P^* (*Weak Pareto for RBSCRs*): For all R^n, con^n, admissible cv^n, S, and x, y: if $x, y \in S$, and $(\forall i)$ Pixy, then $y \notin \mathrm{cf}(S)$; where $\mathrm{cf} = f(R^n, \mathrm{con}^n, \mathrm{cv}^n)$, and f is an RBSCR.

THEOREM 2. *Lib satisfies* U^*, NEC^*, $SCPL^*$ (*and* $WCPL^*$), $SCJL^*$ (*and* $WCJL^*$), *but not* P^* (*or* SP^*).

   Proof. Trivial, given informally in the text.

THEOREM 3. *No RBSCR satisfies* U^*, $WCPL^*$ (*or* $SCPL^*$ *and* NEC^*), *and* P^* (*or* SP^*).

   Proof. Suppose for a reductio that f is an RBSCR that satisfies U^*, $WCPL^*$, and P^*. By U^* and $WCPL^*$ there is some individual, say 1, and some option, say a, of some option set, S, such that: (1) for all R^n and con^n: if $a \notin \mathrm{con}_1(S)$, then $a \notin \mathrm{cf}(S)$, where $\mathrm{cf} = f(R^n, \mathrm{con}^n, \mathrm{cv}^n)$. But suppose: (2) R^n is such that everyone prefers a to all the other options in S, and (3) con^n is such that $a \notin \mathrm{con}_1(S)$. Let $\mathrm{cf} = f(R^n, \mathrm{con}^n, \mathrm{cv}^n)$. Then, by (1) and (3), $a \notin \mathrm{cf}(S)$. And by U^*, P^*, and (2) nothing other than a is in $\mathrm{cf}(S)$. Therefore $\mathrm{cf}(S)$ is empty which contradicts the assumption that f was an RBSCR (which requires cf to be a choice function; never selecting an empty set). Therefore no RBSCR satisfies U^*, $WCPL^*$ (or $SCPL^*$ and NEC^*) and P^* (or SP^*).

THEOREM 4. *No RBSCR satisfies* U^*, $WCJL^*$ (*or* $SCJL^*$), *and* P^* (*or* SP^*).

   Proof (essentially the same as the above except that 1 consents to a, but everyone prefers some option to a). Suppose for a reductio that f is an RBSCR that satisfies U^*, $WCJL^*$, and P^*. By U^* and $WCJL^*$ there is some individual, say 1, and some option, say a, of some option set, S, such that: (1) for all R^n and con^n: if $a \in \mathrm{con}_1(S)$, then $a \in \mathrm{cf}(S)$, where $\mathrm{cf} = f(R^n, \mathrm{con}^n, \mathrm{cv}^n)$. But suppose: (2) R^n is such that everyone prefers some other option in S to a, and (3) con^n is such that $a \in \mathrm{con}_1(S)$. Let $\mathrm{cf} = f(R^n, \mathrm{con}^n, \mathrm{cv}^n)$. Then, by (1) and (3), $a \in \mathrm{cf}(S)$. And by U^*, P^*, and (2) $a \notin \mathrm{cf}(S)$, which gives a contradiction. Therefore no RBSCR satisfies U^*, $WCJL^*$ (or $SCJL^*$), and P^* (or SP^*).

238   PETER VALLENTYNE

Pareto Optimality: $\text{po}(S, R^n) = \{x \in S: {}^{\sim}(\exists y)[y \in S$ & $(\forall i)$ $Riyx$ & $(\exists i)(Piyx)]\}$.

Libertarian Constrained Paretianism (lcp): For all R^n, con^n, admissible cv^n: $\text{lcp}(R^n, \text{con}^n, \text{cv}^n) =$ the choice function, cf, such that for all option sets, S, $\text{cf}(S) = \text{po}(\text{cf}^*(S), R^n)$; where $\text{cf}^* = \text{lib}(R^n, \text{con}^n, \text{cv}^n)$.

RP* (Restricted Weak Pareto): For all R^n, con^n, admissible cv^n, and S: if $x, y \in S$, $x \in \text{cf}(S)$, and $(\forall i)$ $Pixy$, then $y \notin \text{cf}(S)$; where $\text{cf} = f(R^n, \text{con}^n, \text{cv}^n)$, and f is an RBSCR.

RSP* (Restricted Strong Pareto): For all R^n, con^n, admissible cv^n, and S: if $x, y \in S$, $x \in \text{cf}(S)$, $(\forall i)$ $Rixy$, and $(\exists i)$ $Pixy$, then $y \notin \text{cf}(S)$; where $\text{cf} = f(R^n, \text{con}^n, \text{cv}^n)$, and f is an RBSCR.

THEOREM 5. *Lcp satisfies U*, NEC*, SCPL* (and WCPL*), and RSP* (and RP*), but not P* (or SP*).*
   Proof. Trivial, given informally in the text.

SM* (Strong Monotonicity): For all R^n, con^n, admissible cv^n, S, and x and y: if $x, y \in S$ $x \in \text{cf}(S)$, $(\forall i)$ $Riyx$, and $(\exists i)$ $Piyx$, then $y \in \text{cf}(S)$; where $\text{cf} = f(R^n, \text{con}^n, \text{cv}^n)$, and f is an RBSCR.

M* (Weak Monotonicity): For all R^n, con^n, admissible cv^n, S, and x and y: if $x, y \in S$, $x \in \text{cf}(S)$, and $(\forall i)$ $Piyx$, then $y \in \text{cf}(S)$; where $\text{cf} = f(R^n, \text{con}^n, \text{cv}^n)$, and f is an RBSCR.

Libertarian-based Paretianism (lbp): For all R^n, con^n, admissible cv^n: $\text{lbp}(R^n, \text{con}^n, \text{cv}^n) =$ the choice function, cf, such that for all S, $\text{cf}(S) = \{x \in S: [x \in \text{po}(S, R^n)]$ & $[(\forall i)(\exists y)$ $(y \in \text{cf}^*$ & (S) & $Rixy)]\}$, where $\text{cf}^* = \text{lcp}(R^n, \text{con}^n, \text{cv}^n)$.

RPLP* (Reference Point Liberty Protection Condition): For all i, R^n, con^n, admissible cv^n, S, and x: if $(x \in S)$ and $(\forall y)\{[(y \in S)$ & $(\forall j)[(y \notin \text{cv}_j(S)) \vee (y \in \text{con}_j(S))]] \rightarrow Piyx\}$, then $x \notin \text{cf}(S)$; where $\text{cf} = f(R^n, \text{con}^n, \text{cv}^n)$, and f is an RBSCR.

THEOREM 6. *There is an RBSCR – e.g., libertarian-based Paretian-ism – that satisfies U^*, NEC^*, SP^* (and P^*), and $RPLP^*$.*
   Proof. Trivial. Given in text.

NOTES

[1] For a simple proof, see Ch. 6* of Sen (1970), and the appendix of Sen (1976). The theorems stated in these works concern only ordinal collective choice rules, but the proofs are equally valid for all kinds of social choice rules.

[2] See, for example, Gibbard (1973).

[3] The best known contemporary statement of libertarianism is, of course, Nozick (1974).

[4] For generality, consent and constraint violation are assumed to be relative to an option set, since whether an alternative is consented to, or violates the constraints, may depend on what the options are.

[5] I shall assume throughout a version of libertarianism for which it is always possible to avoid violating anyone's rights. For discussion of the issue of ensuring that the rights allocation do not generate empty choice sets, see Wriglesworth (1985), Ch. 2, and Gibbard (1974).

[6] Brian Barry (1986) also makes this point. One might object that the claimed difference between the role of preferences and that of consent in an adequate liberty condition is illusory on the grounds that consent (or lack thereof) is simply revealed preference. I deny, however, that consent (or lack thereof) necessarily reveals a preference of a relevant sort. First, consent can be given (or fail to be given) due to ignorance and mistakes. Such consent does not reveal a preference. Second, the preferences that are relevant for social choice and moral theory may not be *full* preferences. Only non-meddlesome preferences, for example, might be relevant. Although consent is intimately (but not perfectly) connected to full preferences, it is not intimately connected to (for example) non-meddlesome preferences.

[7] Nozick (1974), suggests dealing with Sen's theorem in roughly this manner on pp. 164–166.

[8] These Pareto conditions are similar to the rights constrained Pareto conditions discussed by Austen-Smith (1982). The difference is that where my conditions are conditional on the more preferred alternative being permissible, his conditions are conditional on it satisfying the rights-generated constraints.

[9] If everyone prefers $x3$ to $x2$, why didn't the relevant party consent to $x3$ thereby making it a permissible option? As discussed in Note 6, this might be because the person had a false belief, or simply forgot to give consent; or it might be because the person prefers $x3$ over $x2$ only in terms of her self-regarding preferences, but not in terms of her full preferences (and thus she would not consent).

[10] Unlike most authors on this subject, I have made no mention of meddlesome preferences. This is because I want to emphasize the fact that for liberals liberty depends on one's consent – not on one's preferences. Nonetheless, meddlesome preferences are a serious problem for any preference-based theory. I propose a way of dealing with such preferences in 'The Problem of Unauthorized Welfare', unpublished.

[11] I discuss and defend Rights Based Paretianism in greater detail in Vallentyne (1988).

[12] Work on this paper was partially supported by an internal research grant from the

240   PETER VALLENTYNE

Social Sciences and Humanities Research Council of Canada while I was at The University of Western Ontario. I have benefited from comments from Morry Lipson and Michael Webster.

REFERENCES

Austen-Smith, D.: 1982, 'Restricted Pareto and Rights', *Journal of Economic Theory* **26**, 90–99.

Barry, B.: 1986, 'Lady Chatterly's Lover and Doctor Fischer's Bomb Party', in Jon Elster and Aanund Hylland (eds.), *Foundation of Social Choice Theory*, Cambridge: Cambridge University Press, 1986.

Gibbard, A.: 1973, 'Manipulation of Voting Schemes: A General Result', *Econometrica* **41**, 587–601.

Gibbard, A.: 1974, 'A Pareto-Consistent Pareto Claim', *Journal of Economic Theory* **7**, 388–410.

Nozick, R.: 1974, *Anarchy, State, and Utopia*, New York: Basic Books.

Sen, A. K.: 1970, *Collective Choice and Social Welfare*, San Francisco: Holden-Day.

Sen, A. K.: 1976, 'Liberty, Unanimity, and Rights', *Economica* **43**, 217–246.

Vallentyne, P.: 1988, 'Rights Based Paretianism', *Canadian Journal of Philosophy* **18**, 527–544.

Wriglesworth, J. L.: 1985, *Libertarian Conflicts in Social Choice* (Cambridge: Cambridge University Press).

Department of Philosophy
Virginia Commonwealth University
Richmond, VA 23284–2025, U.S.A.